Quantitative Design of Observational Networks

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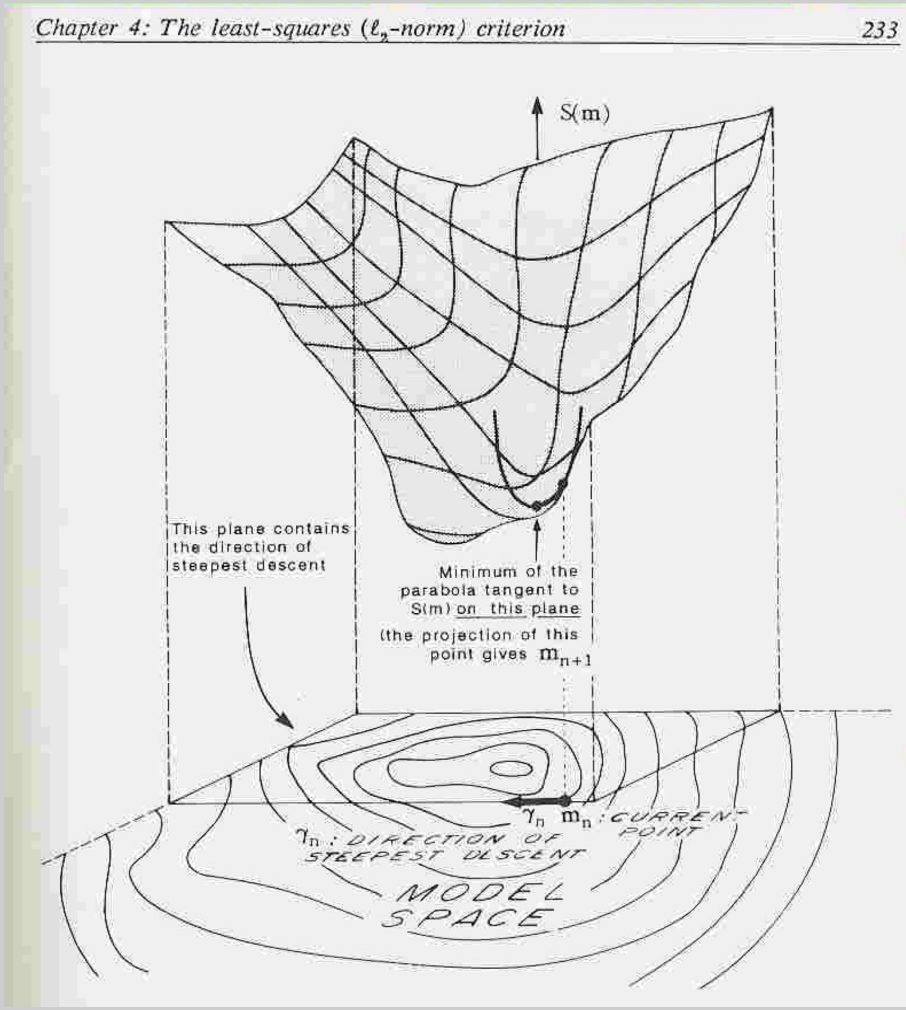
Extended assimilation/network design system



- Iterative minimisation of **J(x)**

$$J(x) = \tfrac{1}{2} \left[(x-x_{pr})^T C_{pr}^{-1}(x-x_{pr}) + (M(x)-d)^T C_d^{-1}(M(x)-d) \right]$$

- Uses gradient of J with respect to parameters
- Second derivatives (Hessian) at minimum $\mathbf{x}_{po}$ provide approximation of parameter uncertainties (error bars)

$$C_{po}^{-1} \approx d^2J(x_{po}) / dx^2$$

- Uncertainties on current of future target quantities (fresh water flux (MOC) or ice pressure on a boat in the northern sea route) via linearisation of model (Jacobian matrix)

$$C_{NEP} = dM/ dx \; C_{po} \; dM/ dx^T$$

- All derivatives provided via automatic differentiation of model code (TAF), see Kaminski et al. (2003)
- Figure taken from Tarantola (1987)

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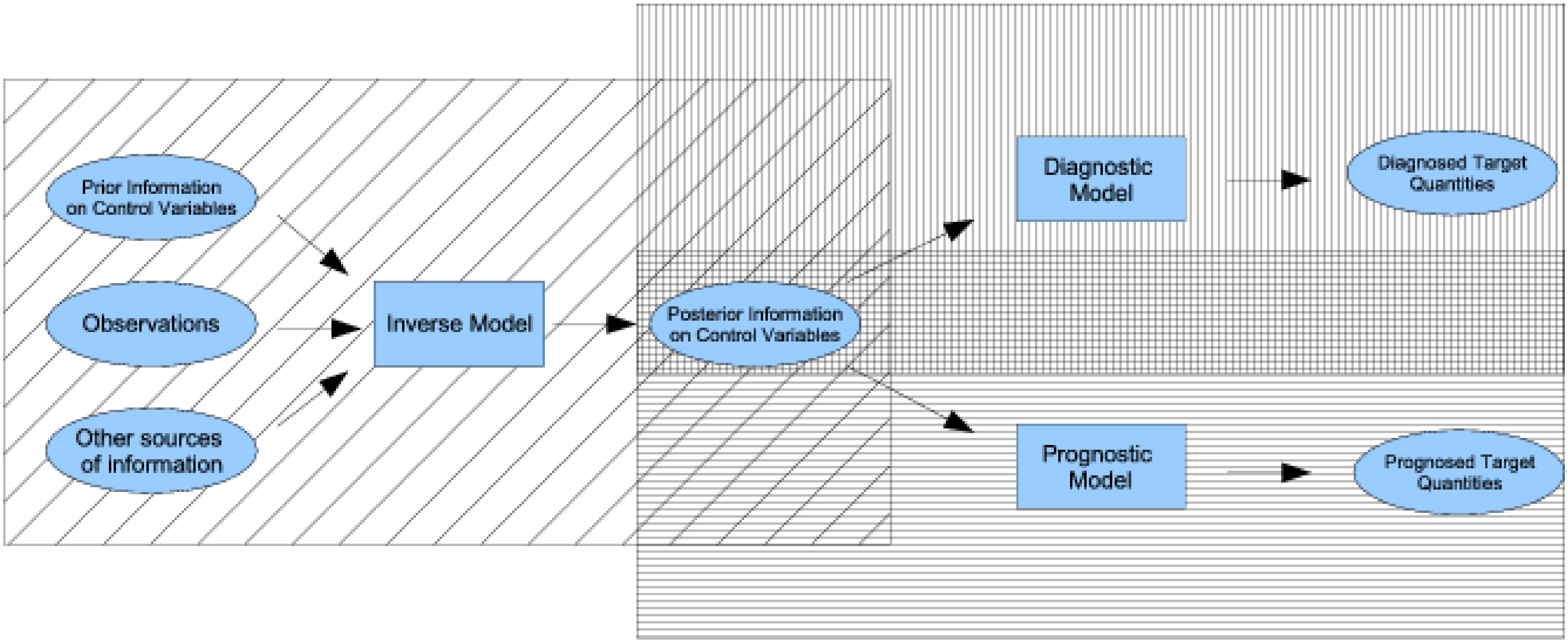
CCDAS scheme



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Uncertainty for target in 2 steps

x: Parameters

x_{pr}: Priors

C_{pr}: Uncertainties

$M(x)$: Model

d: Observations

C_d: Their uncertainties

σ_{d_i}: Uncorrelated!

$J(x)$: Cost function

$\frac{d^2 J(x)}{dx^2}$: Hessian

x_{po}: Posterior parameters

C_{po}: Posterior uncertainties

$y(x)$: Target quantity

σ_y: Its uncertainty

$$J(x) = \frac{1}{2}(x - x_{pr})^T {C_{pr}}^{-1} (x - x_{pr}) + \frac{1}{2} \sum_{i=1,nd} \left(\frac{M_i(x) - d_i}{\sigma_{d_i}} \right)^2$$

$$\frac{d^2 J(x)}{dx^2} = {C_{pr}}^{-1} + \sum_{i=1,nd} \frac{1}{\sigma^2_{d_i}} \frac{d^2}{dx^2} (M_i(x) - d_i)^2$$

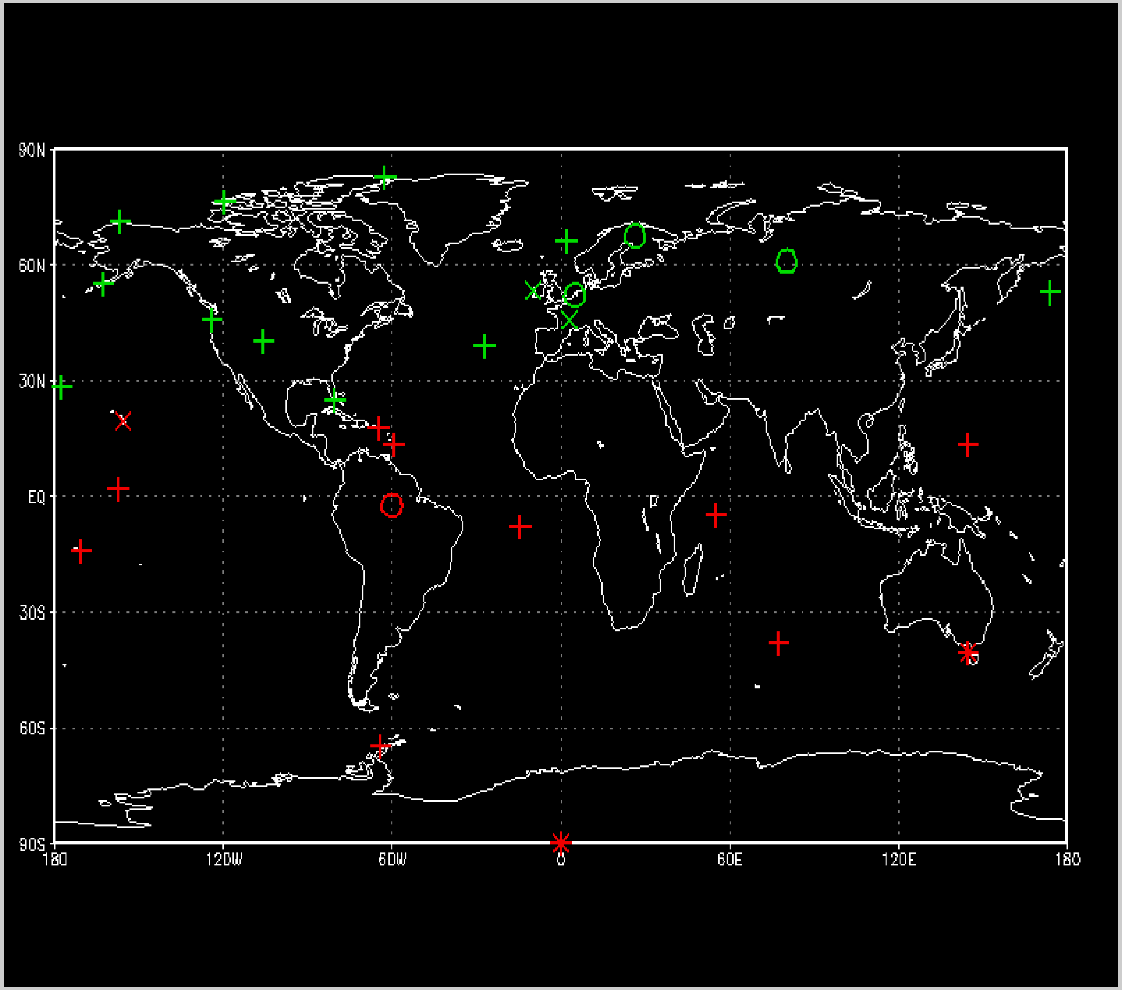
- Hessian independent of x for linear model
- For synthetic data use $d = M(x)$.
- Decomposes nicely, can precompute model contribution

$$C_{po} \approx \frac{d^2 J(x_{po})}{dx^2}^{-1}$$

$$\sigma_y \approx \frac{dy(x_{po})}{dx} C_{po} \frac{dy(x_{po})}{dx}^T \approx \frac{dy(x_{po})}{dx} \frac{d^2 J(x_{po})}{dx^2}^{-1} \frac{dy(x_{po})}{dx}^T$$

Derivative information can be efficiently provided by compiler tool TAF

Sketch of Network Designer



Observations [sigma]

+ Flask [enter]
x Continuous [enter]
o Eddy Flux [enter]

|Compute|

Targets [sigma]

European Uptake [ ]
Global Uptake [ ]

Assumptions and Ingredients

Assumptions:

- Gaussian uncertainties on priors, observations, and from model error (or function of Gaussian, e.g. lognormal)
- Model not too non linear
- What else?

Ingredients:

- Ability to estimate uncertainties for priors, observations and due to model error; requires expertise of observationalists and modellers
- Assimilation system that can (efficiently) propagate uncertainties; helpful: adjoint, Hessian, and Jacobian codes, NAOSIMDAS could be the core
- Need to take logistic constraints into account