# Quantitative Design of Observational Networks

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# Motivation

Questionnaire

The intention of this questionnaire is to stimulate the discussion during the meeting.

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4. Observational Network design and modelling

a) Are state-of-the-art Arctic models able to assist in the design of observational networks. If not, what is needed?

# Motivation

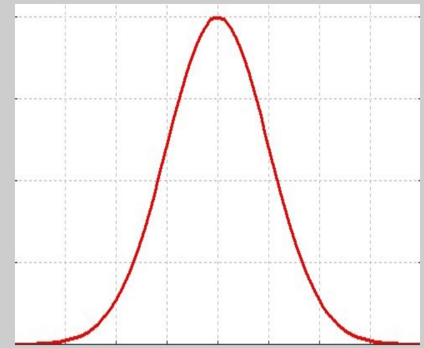
Can construct a machinery that,
for a given network and a given target quantity,
can approximate the uncertainty
with which the value of the target quantity is constrained by the observations

# Outline

- Motivation
- Method
- Demo
- Which assumptions?
- Links to further information
- Discussion
-> Any potential show stoppers for application to Arctic?

# Model and Observational Uncertainties

- No observation/no model is perfect.
- It is convenient to quantify observations and their model counterpart by probability density functions PDFs.
- The simplest assumption is that they are Gaussian.



$$\sigma_d^2 = \sigma_{obs}^2 + \sigma_{mod}^2$$

total uncertainty

uncertainty from observational error

uncertainty from model error

- If the observation refers to a point in space and time, there is a representation error because the counterpart simulated by the model refers to a box in space and time.
- The corresponding uncertainty must be accounted for either by the observational or by the model contribution to total uncertainty.

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# Posterior Uncertainty

If the model was linear:

$$J(\tilde{\mathbf{x}}) = \frac{1}{2}\left[(\mathbf{M}\tilde{\mathbf{x}} - \mathbf{d})^T \mathbf{C}(d)^{-1}(\mathbf{M}\tilde{\mathbf{x}} - \mathbf{d}) + (\tilde{\mathbf{x}} - \mathbf{x_0})^T \mathbf{C}(x_0)^{-1}(\tilde{\mathbf{x}} - \mathbf{x_0})\right]$$

and data + priors have Gaussian PDF, then the posterior PDF is also Gaussian: $\rho(x) \sim e^{-J(x)}$

with mean value:

$$\mathbf{x} = \mathbf{x_0} + [\mathbf{M}^T \mathbf{C}(d)^{-1}\mathbf{M} + \mathbf{C}(x_0)^{-1}]^{-1}\mathbf{M}^T \mathbf{C}(d)^{-1}(\mathbf{d} - \mathbf{M}\mathbf{x_0})$$

and uncertainty:

$$\mathbf{C}(x)^{-1} = \mathbf{M}^T \mathbf{C}(d)^{-1}\mathbf{M} + \mathbf{C}(x_0)^{-1}$$

which are related to the Hessian of the cost function:

$$\mathbf{C}(x)^{-1} = \mathbf{H} \leftarrow \frac{\partial^2 J}{\partial x_i \partial x_j}$$

For a non-linear model, this is an approximation

# Uncertainty for target in 2 steps

$x$: Parameters

$x_{pr}$: Priors

$C_{pr}$: Uncertainties

$M(x)$: Model

$d$: Observations

$C_d$: Their uncertainties

$\sigma_{d_i}$: Uncorrelated!

$J(x)$: Cost function

$\frac{d^2 J(x)}{dx^2}$: Hessian

$x_{po}$: Posterior parameters

$C_{po}$: Posterior uncertainties

$y(x)$: Target quantity

$\sigma_y$: Its uncertainty

$$J(x) = \frac{1}{2}(x - x_{pr})^T {C_{pr}}^{-1} (x - x_{pr}) + \frac{1}{2} \sum_{i=1,nd} \left( \frac{M_i(x) - d_i}{\sigma_{d_i}} \right)^2$$

$$\frac{d^2 J(x)}{dx^2} = {C_{pr}}^{-1} + \sum_{i=1,nd} \frac{1}{\sigma_{d_i}^2} \frac{d^2}{dx^2} (M_i(x) - d_i)^2$$

- Hessian independent of $x$ for linear model
- For synthetic data use $d = M(x)$.
- Decomposes nicely, can precompute model contribution

$$C_{po} \approx \frac{d^2 J(x_{po})}{dx^2}^{-1}$$

$$\sigma_y \approx \frac{dy(x_{po})}{dx} C_{po} \frac{dy(x_{po})}{dx}^T \approx \frac{dy(x_{po})}{dx} \frac{d^2 J(x_{po})}{dx^2}^{-1} \frac{dy(x_{po})}{dx}^T$$

Derivative information can be efficiently provided by compiler tool TAF

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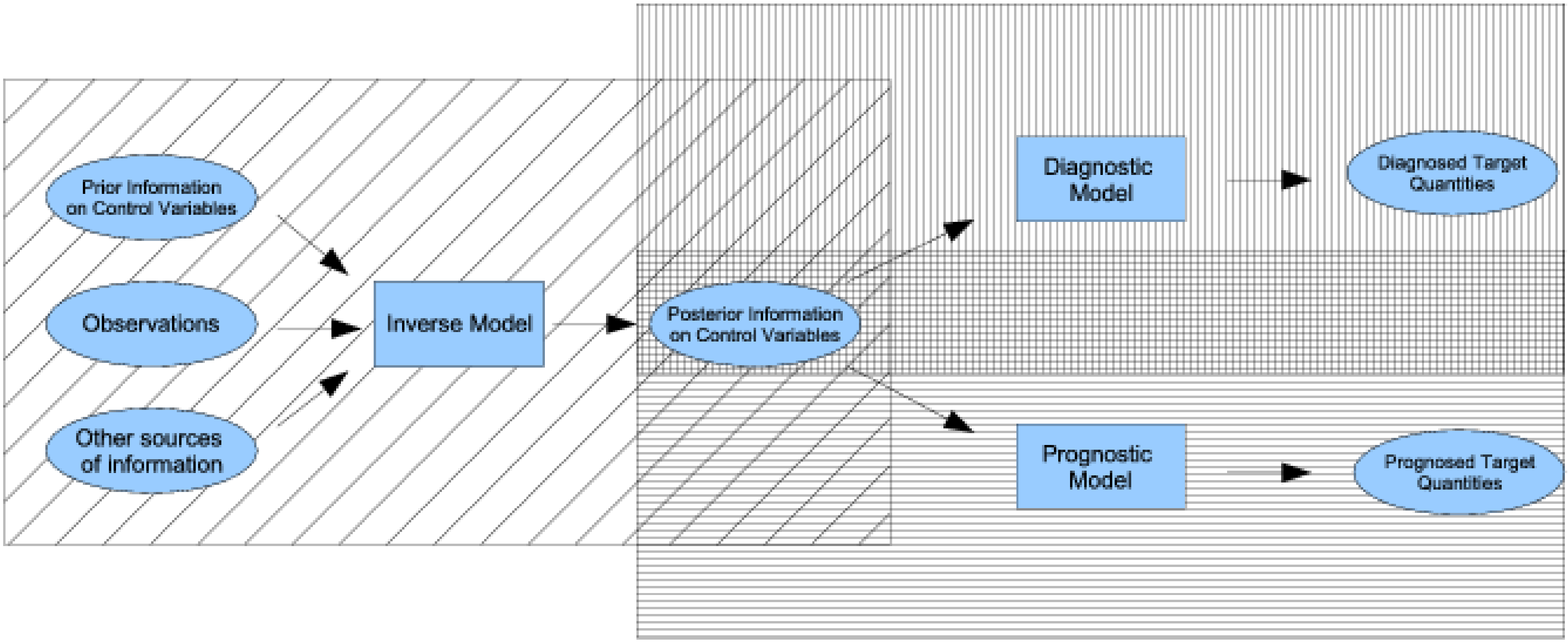
# CCDAS scheme

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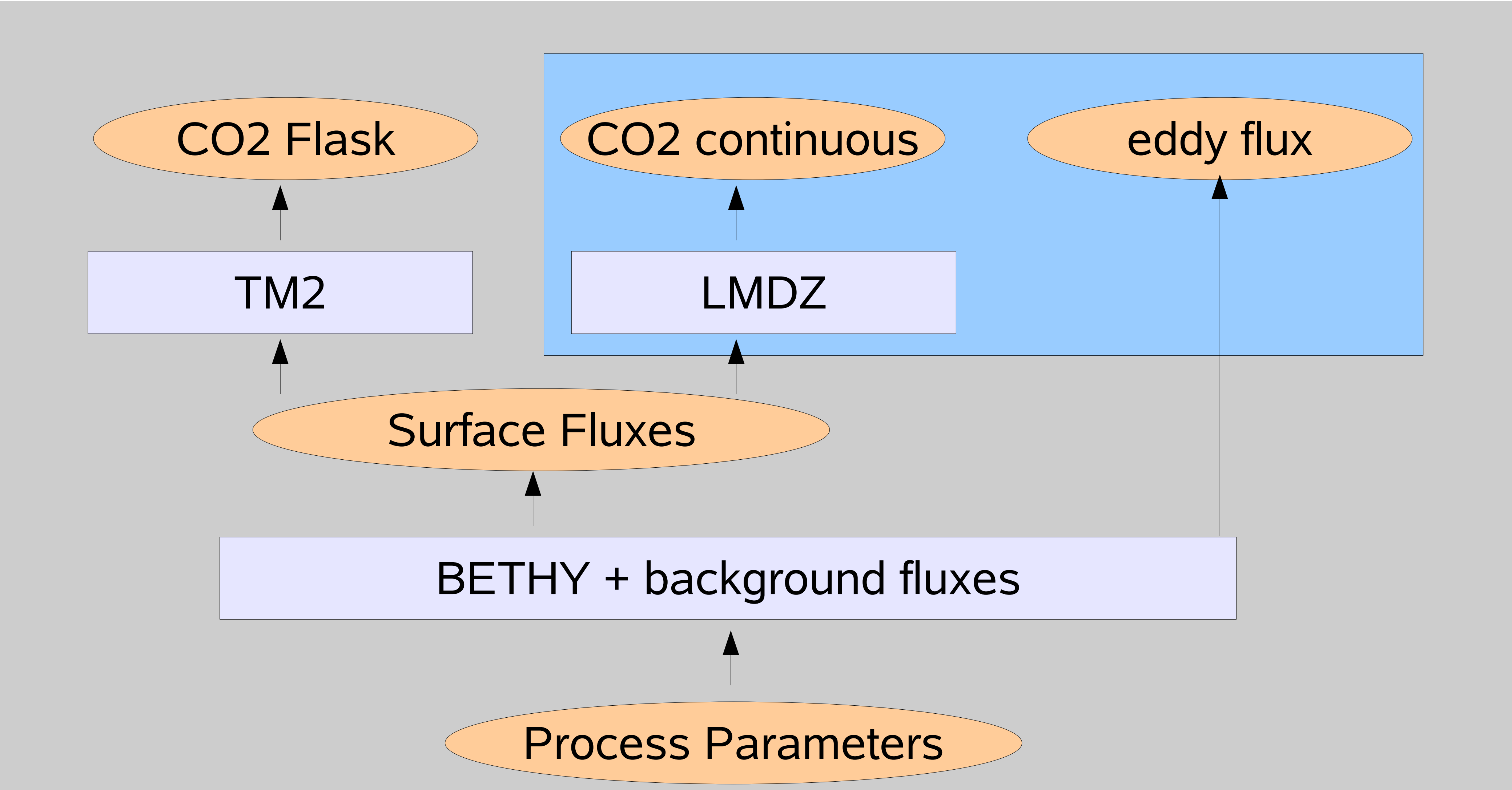
# Carbon Cycle Data Assimilation System (CCDAS) Forward Modelling Chain

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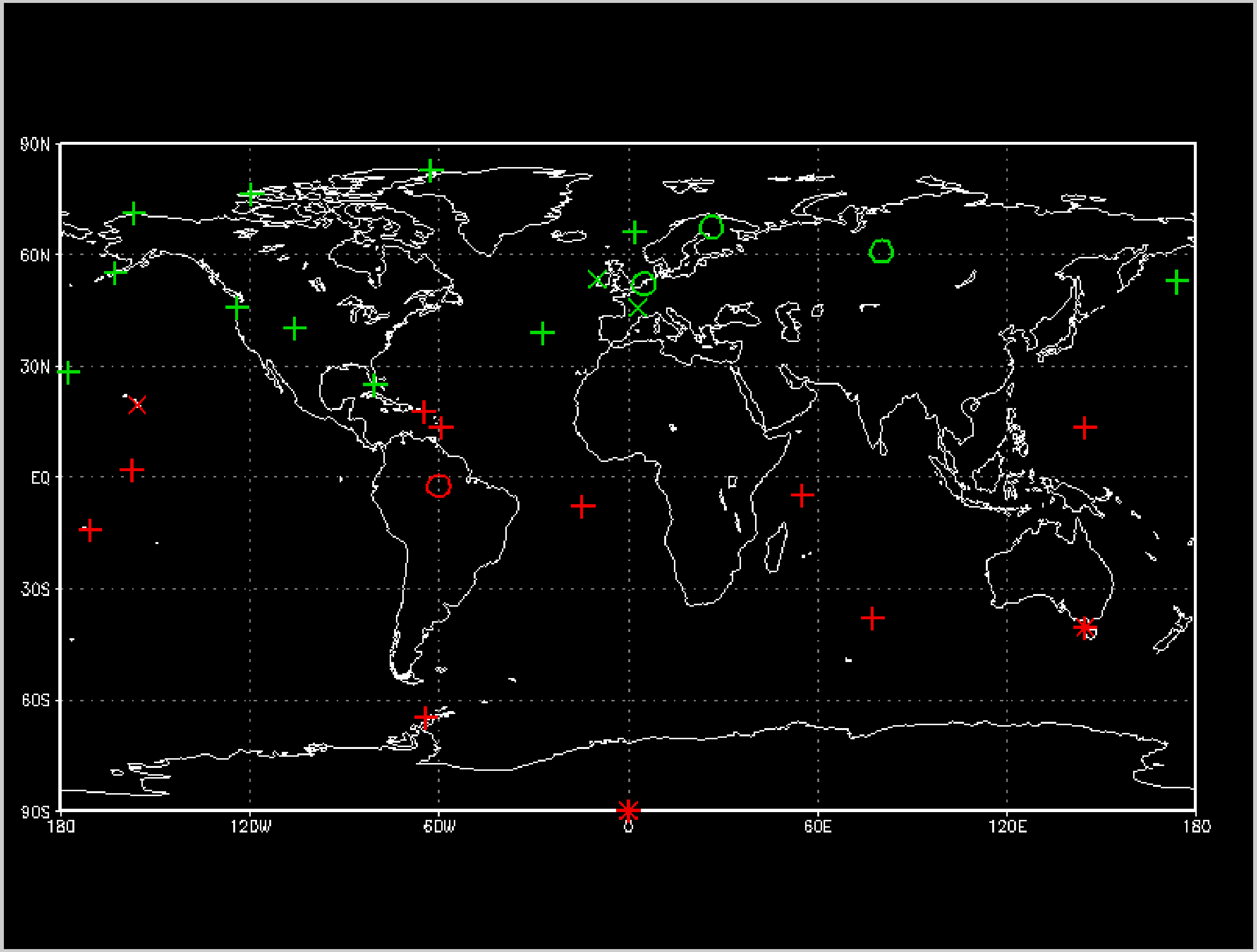
# Sketch of Network Designer




Observations [sigma]

+ Flask [enter]
x Continuous [enter]
o Eddy Flux [enter]

|Compute|

Targets [sigma]

European Uptake [ ]
Global Uptake [ ]

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# Assumptions and Ingredients

Assumptions:

•Gaussian uncertainties on priors, observations, and from model error (or function of Gaussian, e.g. lognormal)

•Model not too non linear

•What else?

Ingredients:

•Ability to estimate uncertainties for priors, observations and due to model error; requires expertise of observationalists and modellers

•Assimilation system that can (efficiently) propagate uncertainties; helpful: adjoint, Hessian, and Jacobian codes,
NAOSIMDAS could be the core

•Need to take logistic constraints into account

# Further Information

Terrestrial assimilation system applications and papers:
http://CCDAS.org

The corresponding network design project:

http://IMECC.CCDAS.org

with link to paper on network design (Kaminski and Rayner, in press)

Assimilation in the Arctic:

http://www.damocles-eu.org

More assimilation systems, applications and papers:

http://FastOpt.com

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