

# An overview on adjoint applications in Earth System Modelling

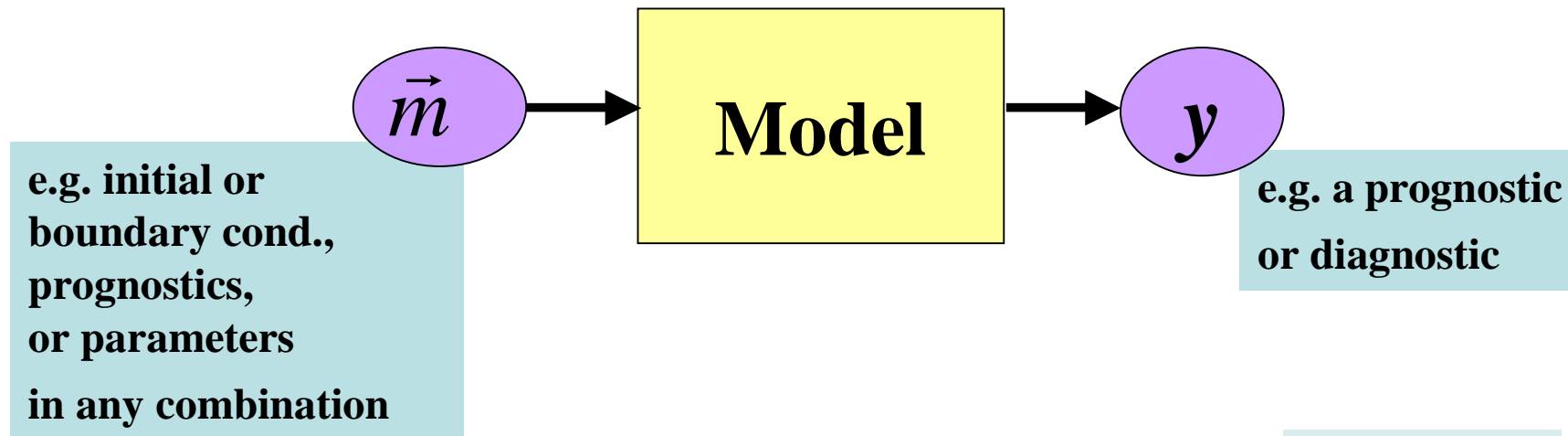
*R. Giering<sup>1</sup>, T. Kaminski<sup>1</sup>, M. Scholze<sup>2</sup>,  
P. Rayner<sup>3</sup>, W. Knorr<sup>4</sup>, H. Widmann<sup>4</sup>,  
R. Todling<sup>5</sup>, S.-J. Lin<sup>5</sup>*

# Overview

- **Sensitivities**
- **Data Assimilation**
- **Uncertainty propagation**
- **Summary, Links**

# Sensitivities

Think of your model as a function:

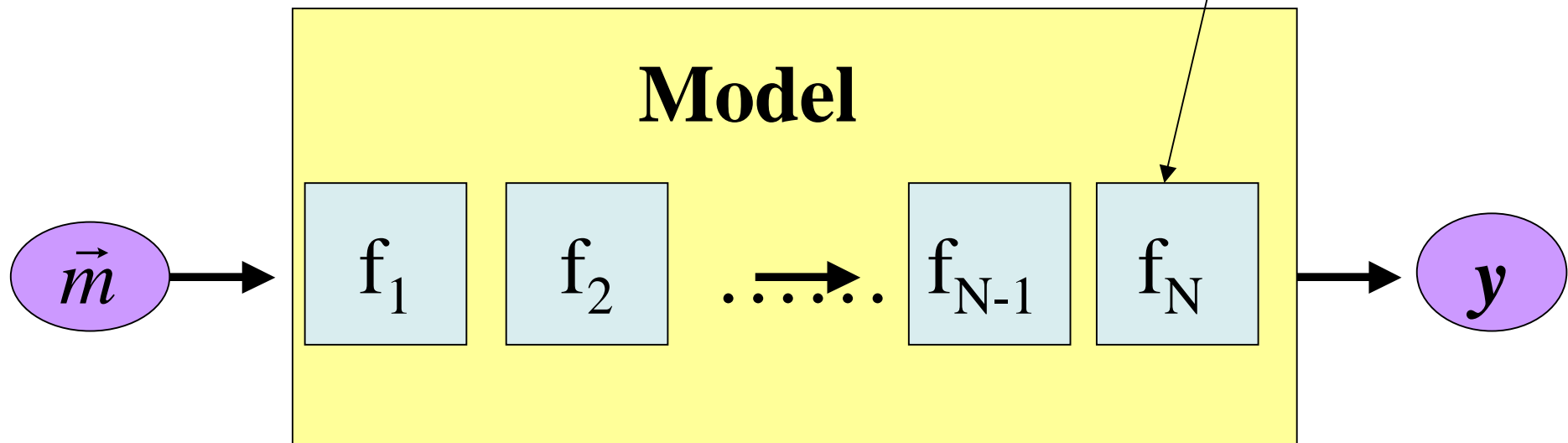


Sensitivity is the derivative (gradient) of that function:

$$\nabla y$$

# Sensitivities

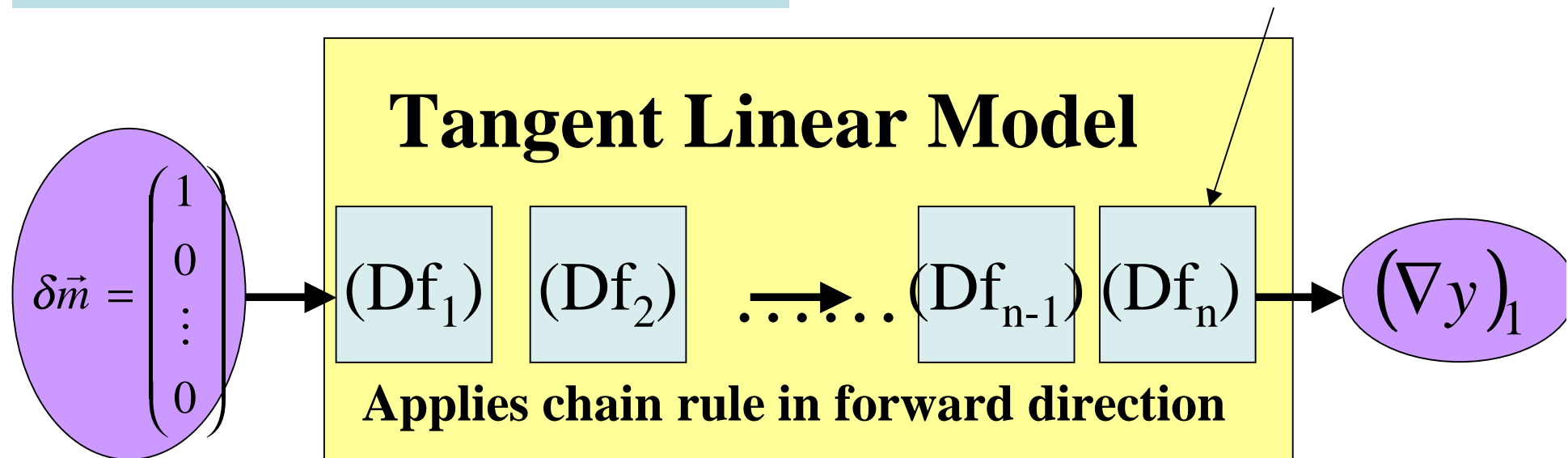
Statements in code  
define elementary functions  
such as +, /, \*\*, sin, exp ...



# Sensitivities by Automatic Differentiation

Cost for full derivative  
proportional to size of  $m$ !

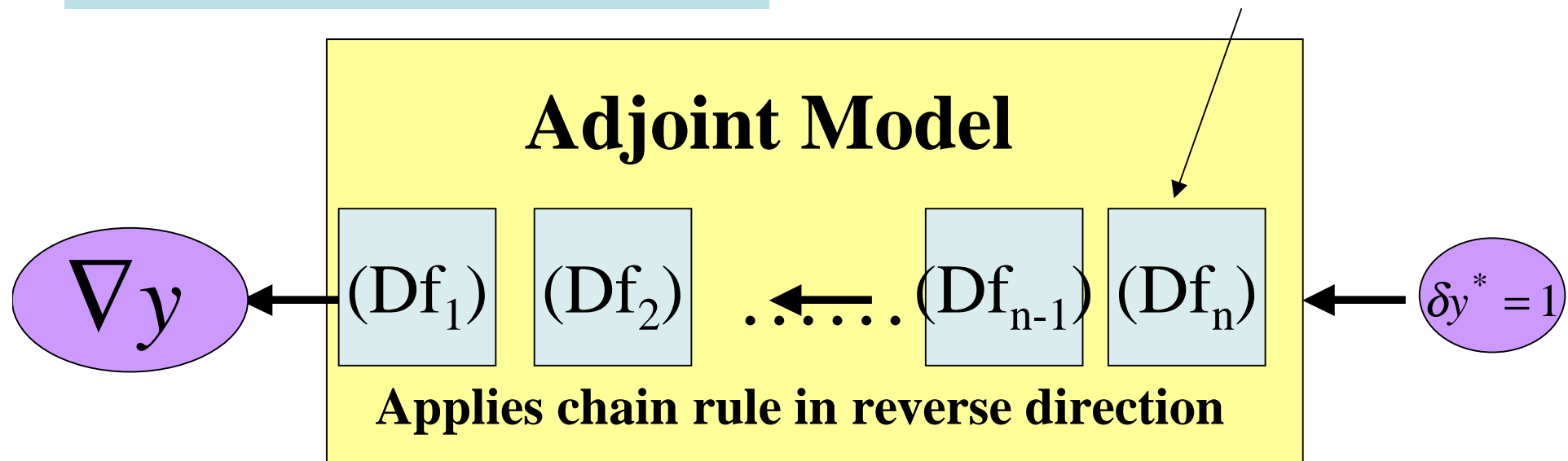
Derivatives of elementary  
functions are simple,  
they yield local Jacobians



# Adjoint Sensitivities

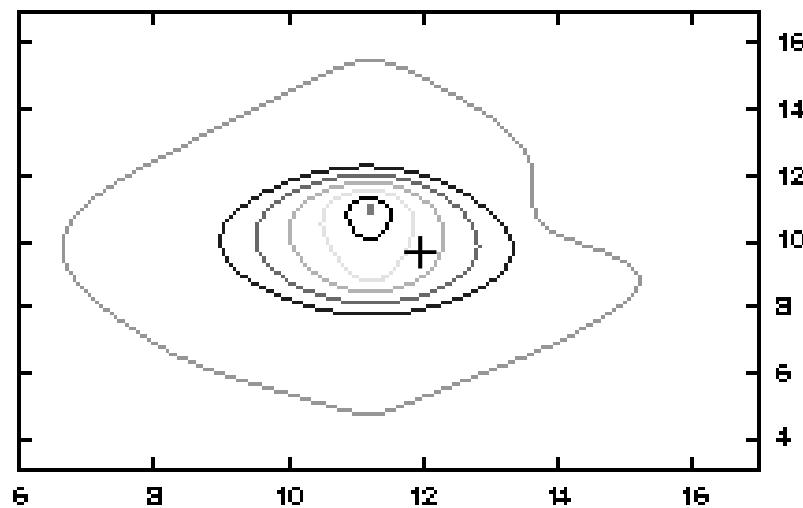
Cost is independent  
of size of  $m$ !

Derivatives of elementary  
functions are simple,  
they yield local Jacobians



# Sensitivities of an AGCM

Sensitivity of potential temperature to zonal velocity  
pt(12,10.0) to u25(±10)

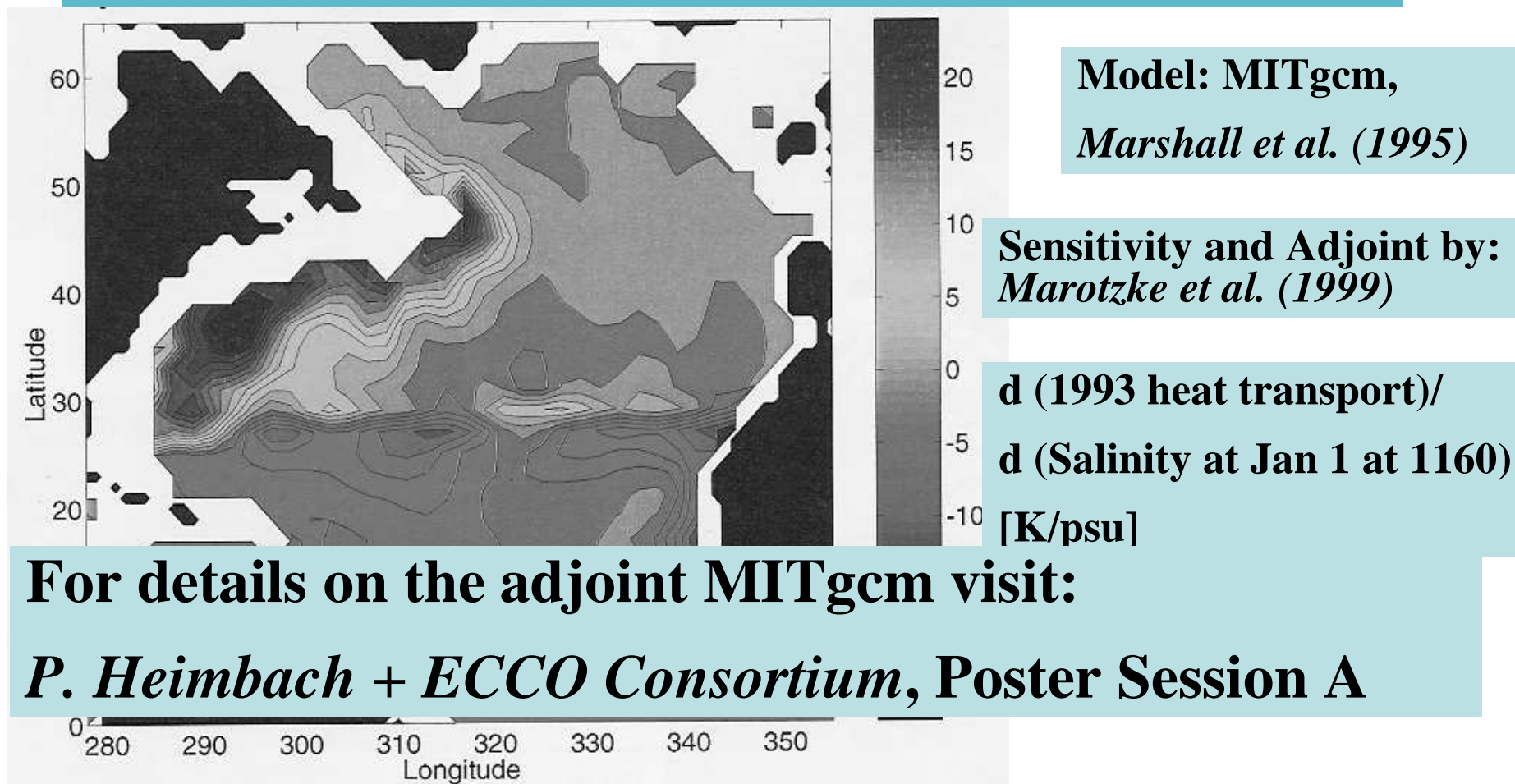


-1e-05  
-2e-05  
-3e-05  
-4e-05  
-5e-05  
-6e-05  
-7e-05

$d\theta/d$  (initial u 1 lev above) [K/(m/s)]

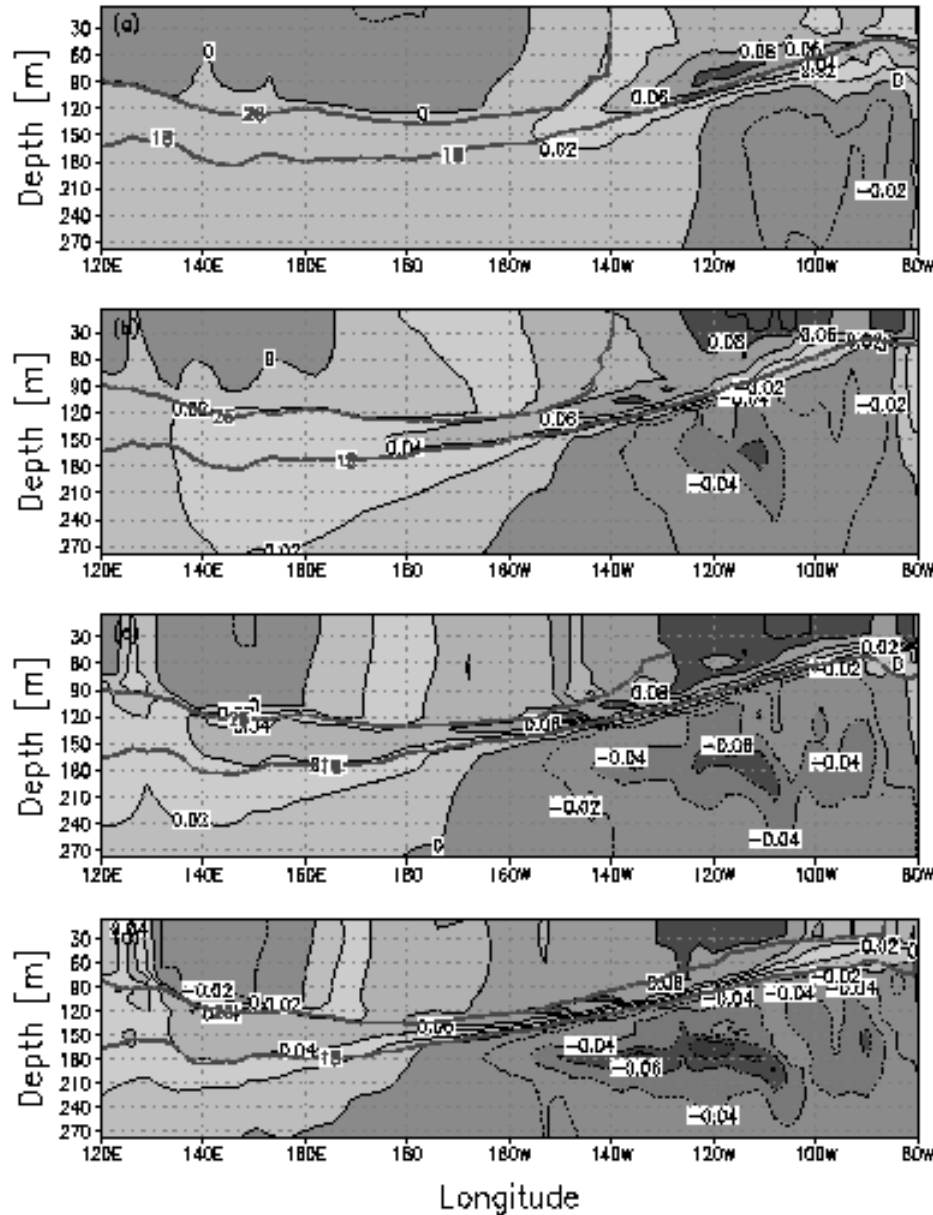
Model: dyn. core of NASA fvGCM  
(*Lin and Rood, 96/97, Lin 97*)

# Sensitivities of an OGCM





# Sensitivities of OGCM + stat. Atmosphere (HCM)

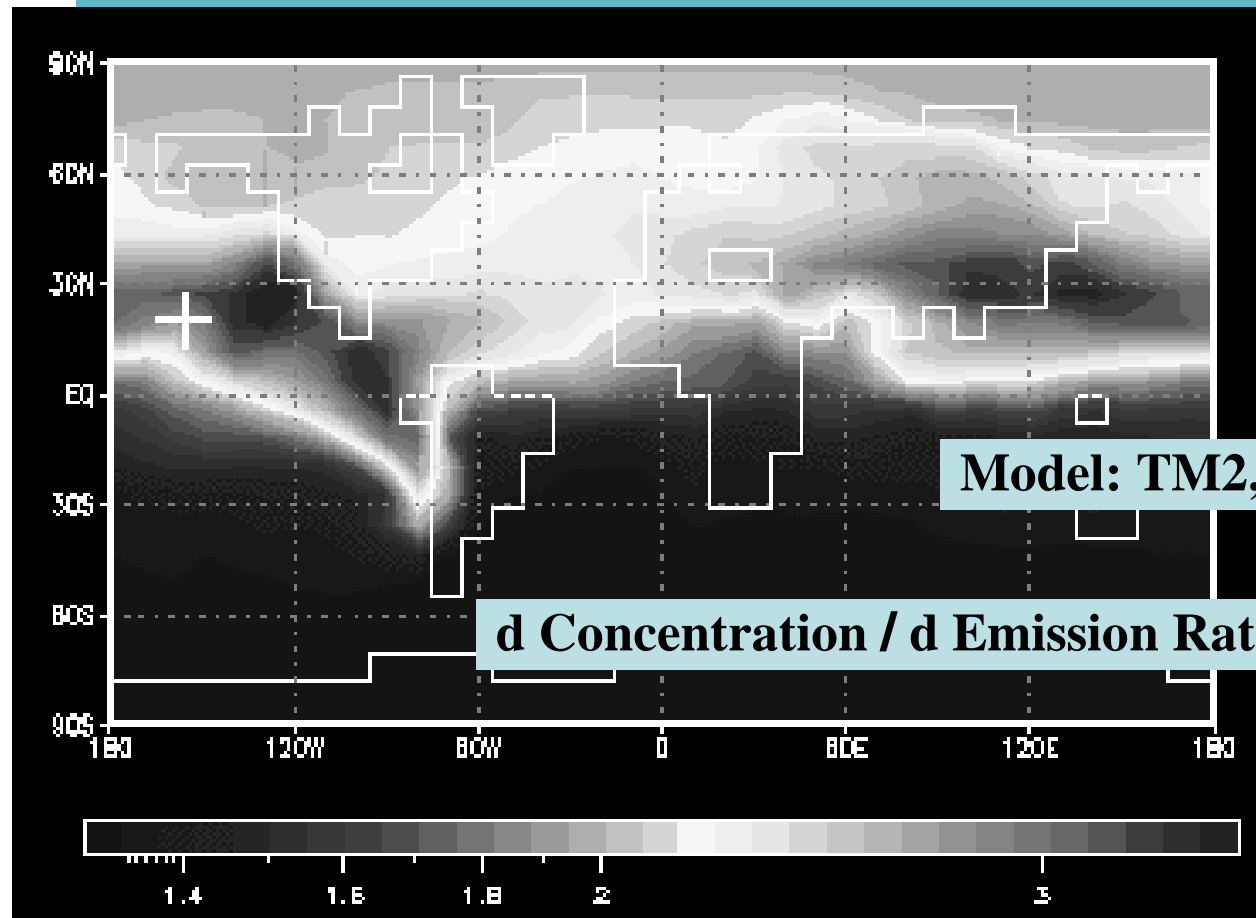


Model: MOM3,  
*Pacanowski and Griffes (1999)*

Adjoint and Sensitivity by:  
*Galanti et al. (2002)*

$d(T \text{ around } 100W, 60m \text{ at Dec } 15)/dT$   
With lags: 1,3,5,7 month,  
[no dimension]

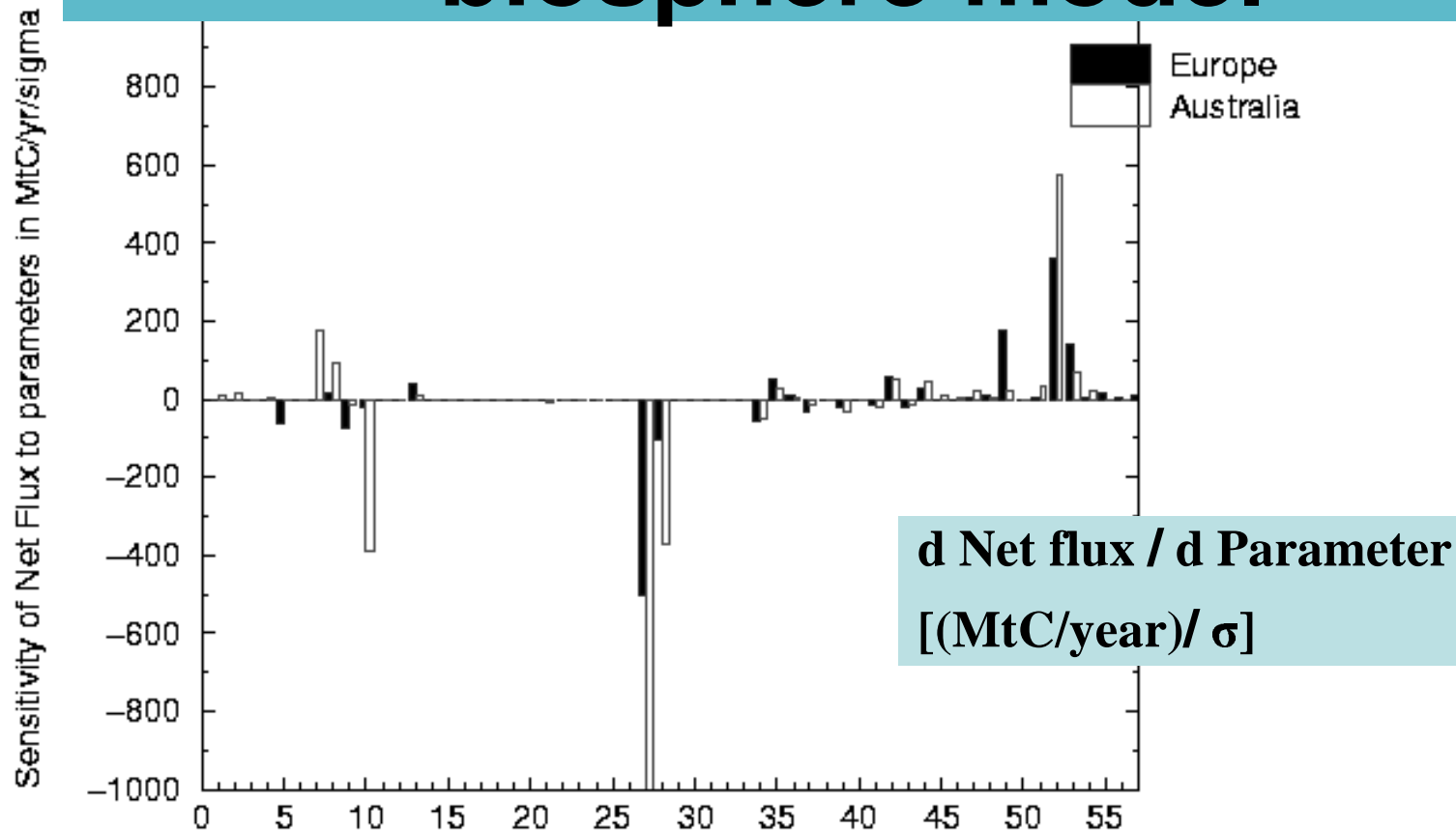
# Sensitivities of a CTM



Model: TM2, Heimann (1996)

$\frac{d \text{ Concentration}}{d \text{ Emission Rate}}$  [ppmv/(Pg/year)]

# Sensitivities of a terrestrial biosphere model



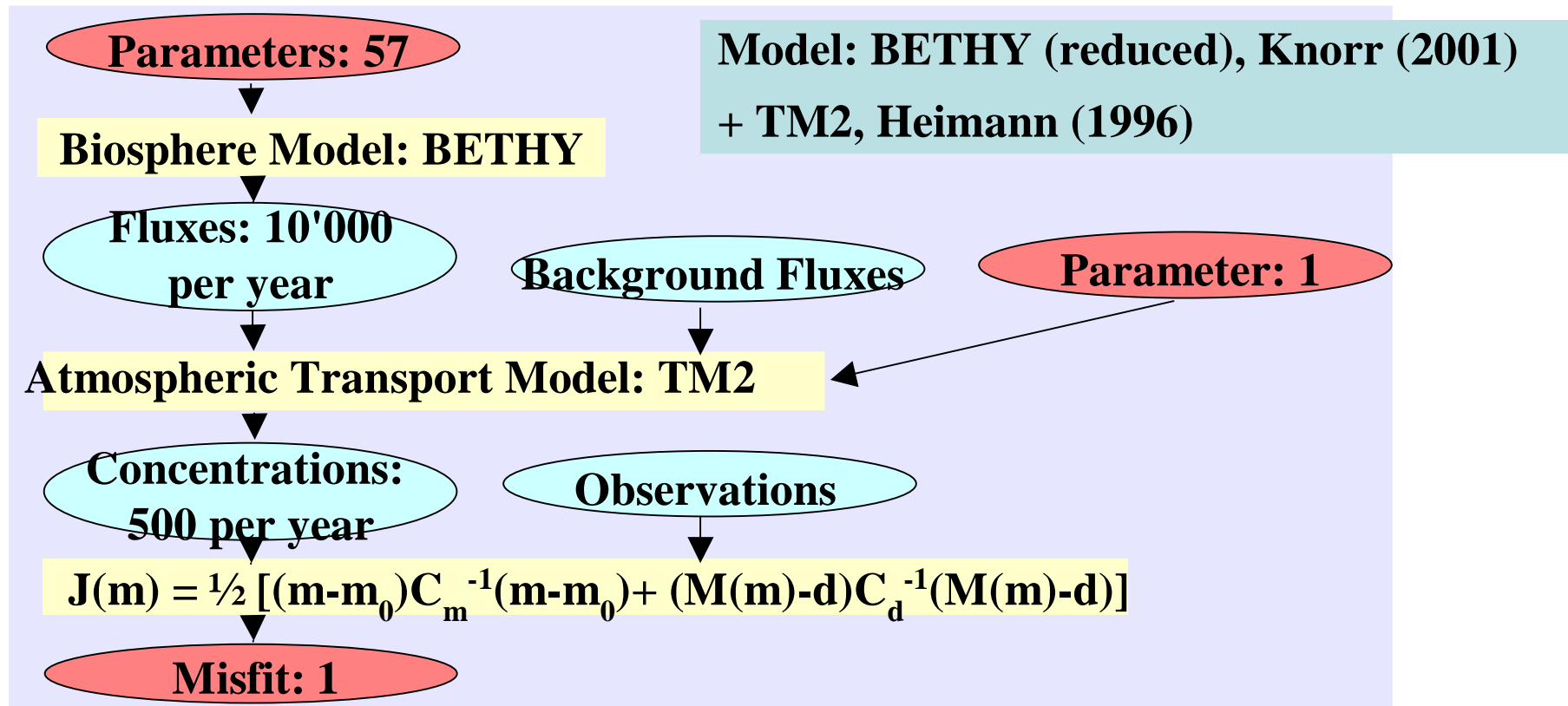
Model: reduced version of BETHY, Knorr (2001)

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# Calibration of biosphere model

within Carbon Cycle Data Assimilation System (CCDAS)



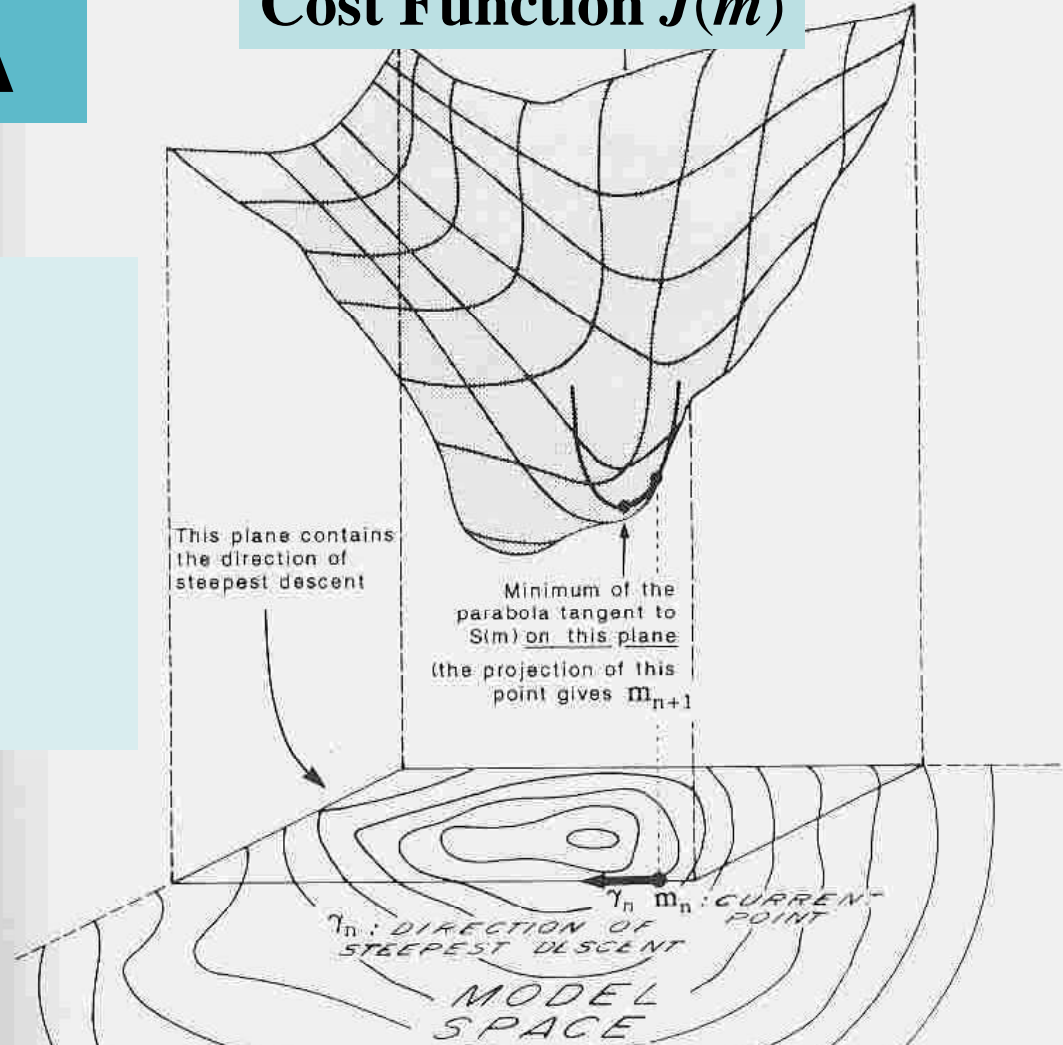
\* **ocean:** Takahashi et al. (1999), LeQuere et al. (2000); **emissions:** Marland et al. (2001), Andres et al. (1996); **land use:** Houghton et al. (1990)

# Adjoint Method/ Variational DA

Gradient of  $J(m)$  w.r.t.  $m$   
points uphill !

$$\nabla J(m)$$

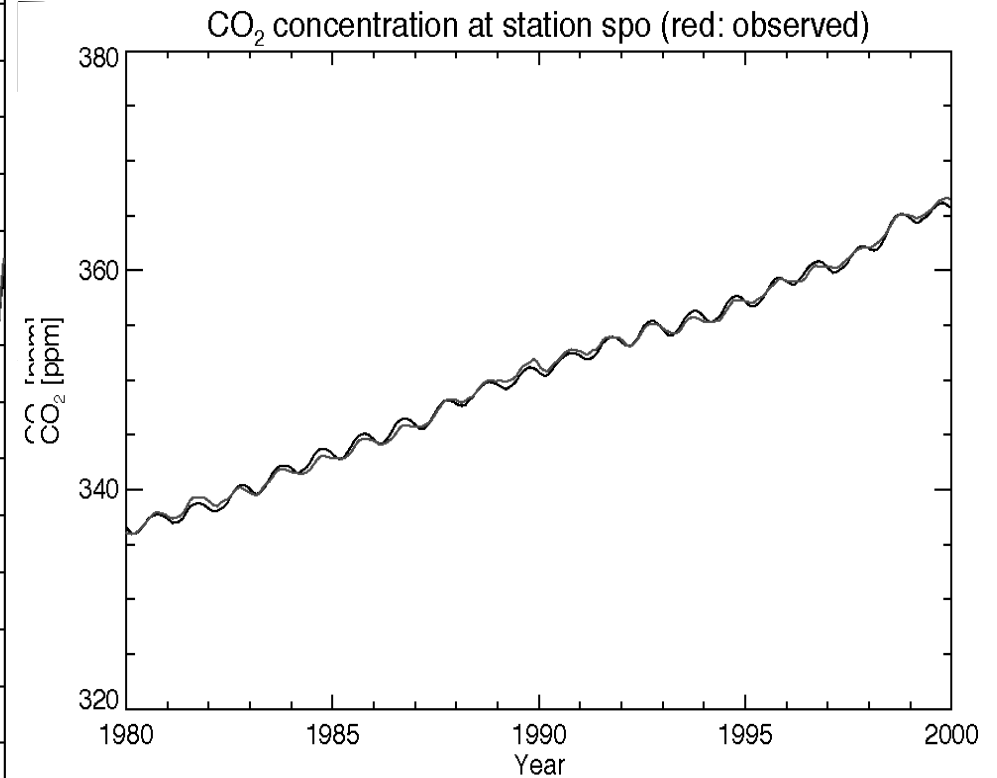
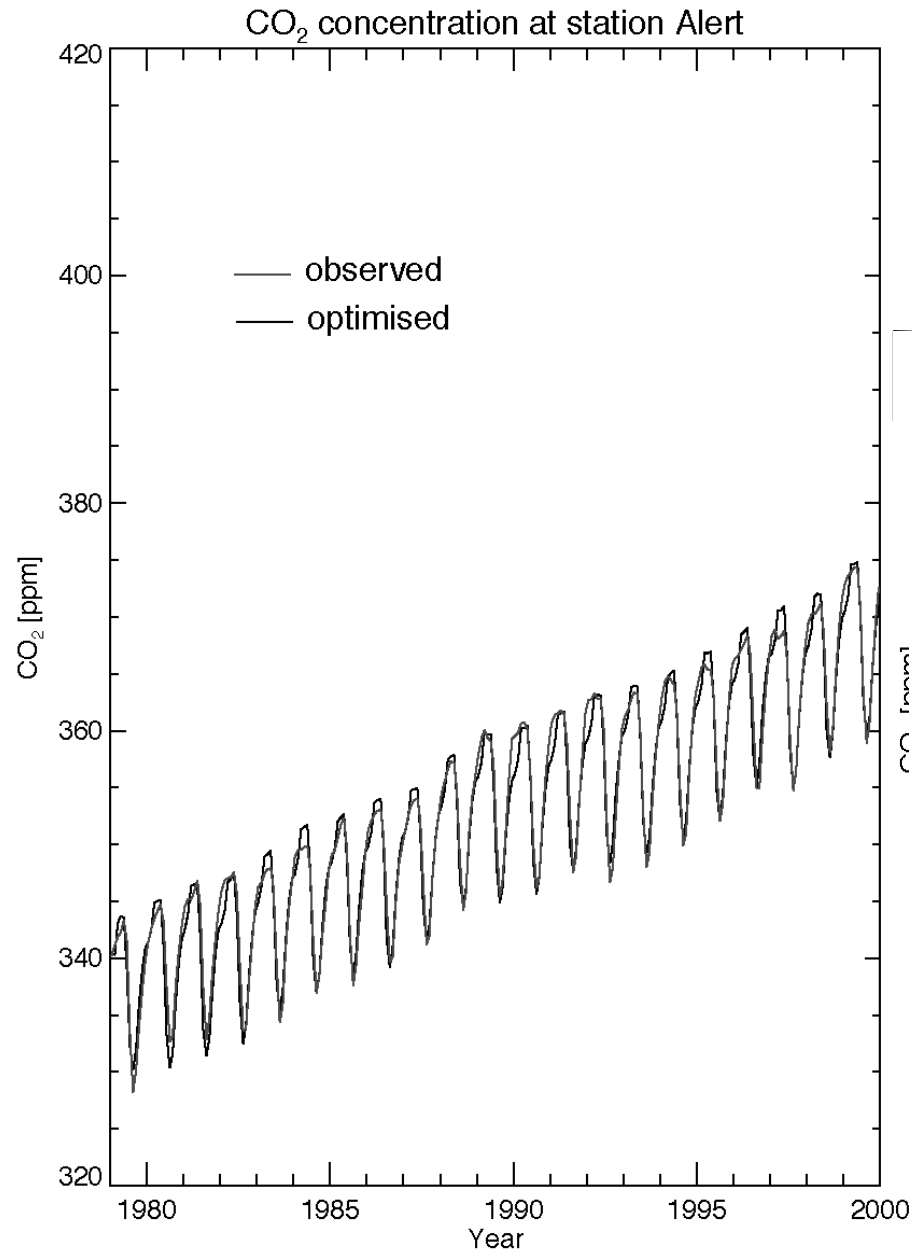
## Cost Function $J(m)$



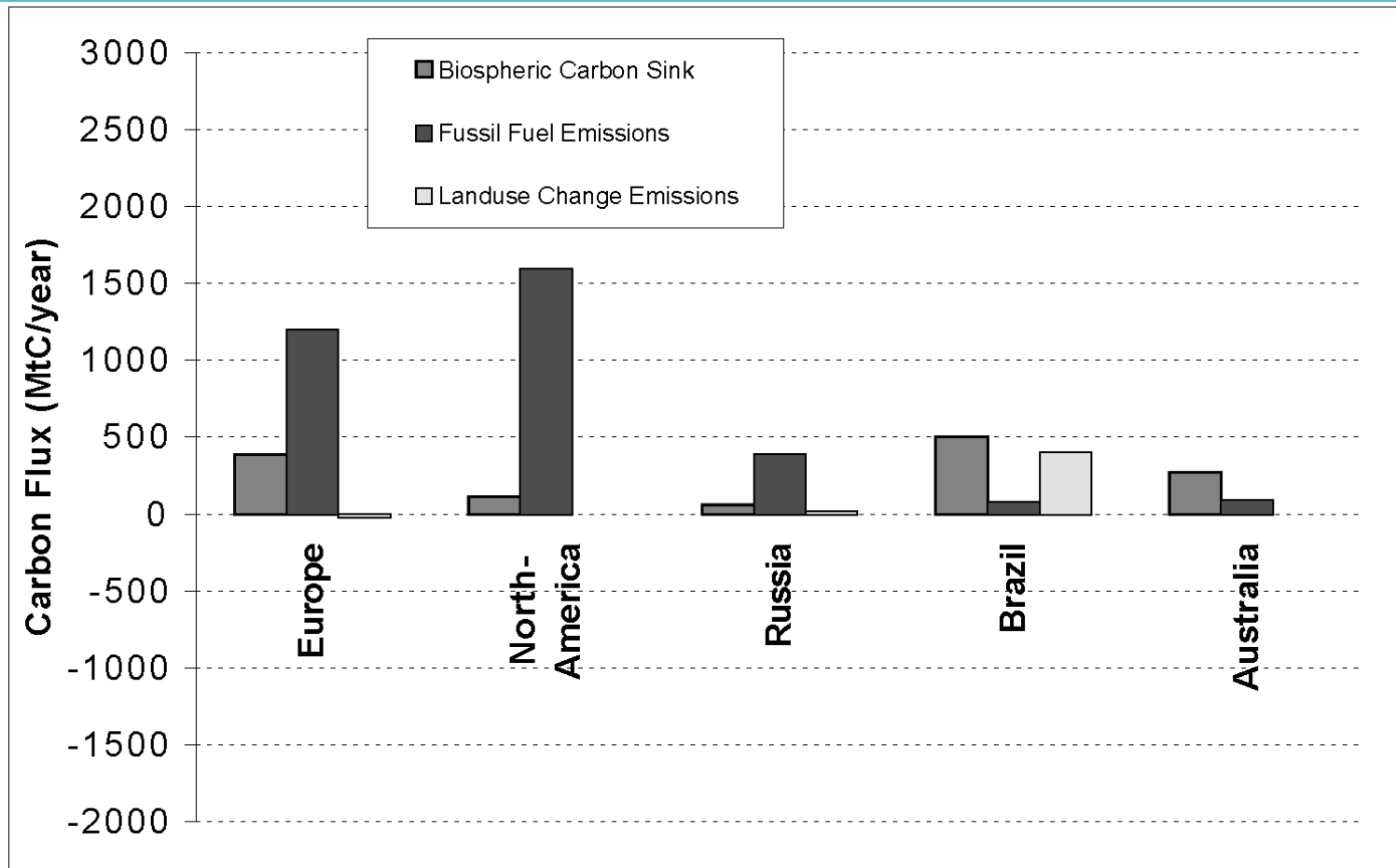
## Space of $m$ (model parameters)

Figure taken from  
Tarantola '87

# Optimisation CCDAS



# Predictions of calibrated model CCDAS





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# Covariances of Parameter-Uncertainties

Second Derivative (Hessian) of  $J(m)$  yields curvature of  $J$ , provides estimated uncertainty in  $m_{opt}$

$$C_m \approx \left\{ \frac{\partial^2 J(m_{opt})}{\partial m_{i,j}^2} \right\}^{-1}$$

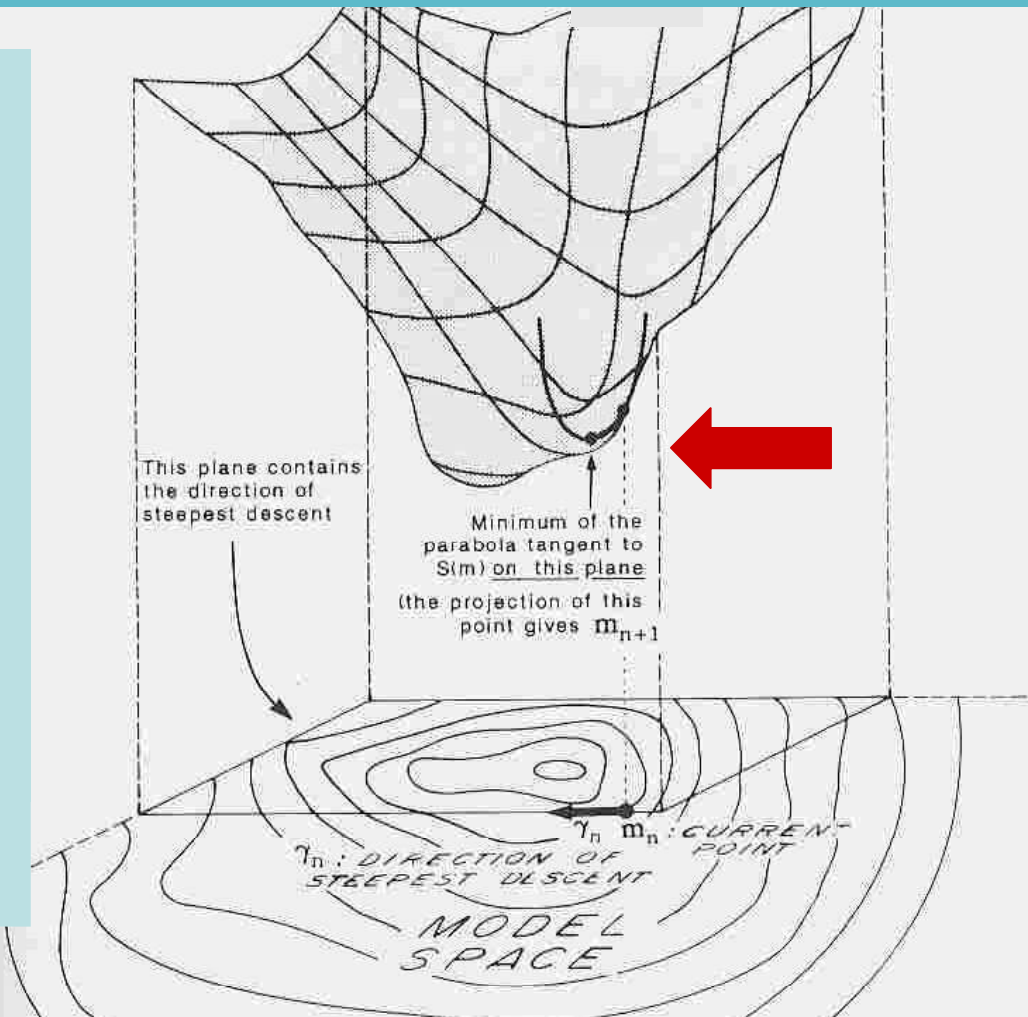


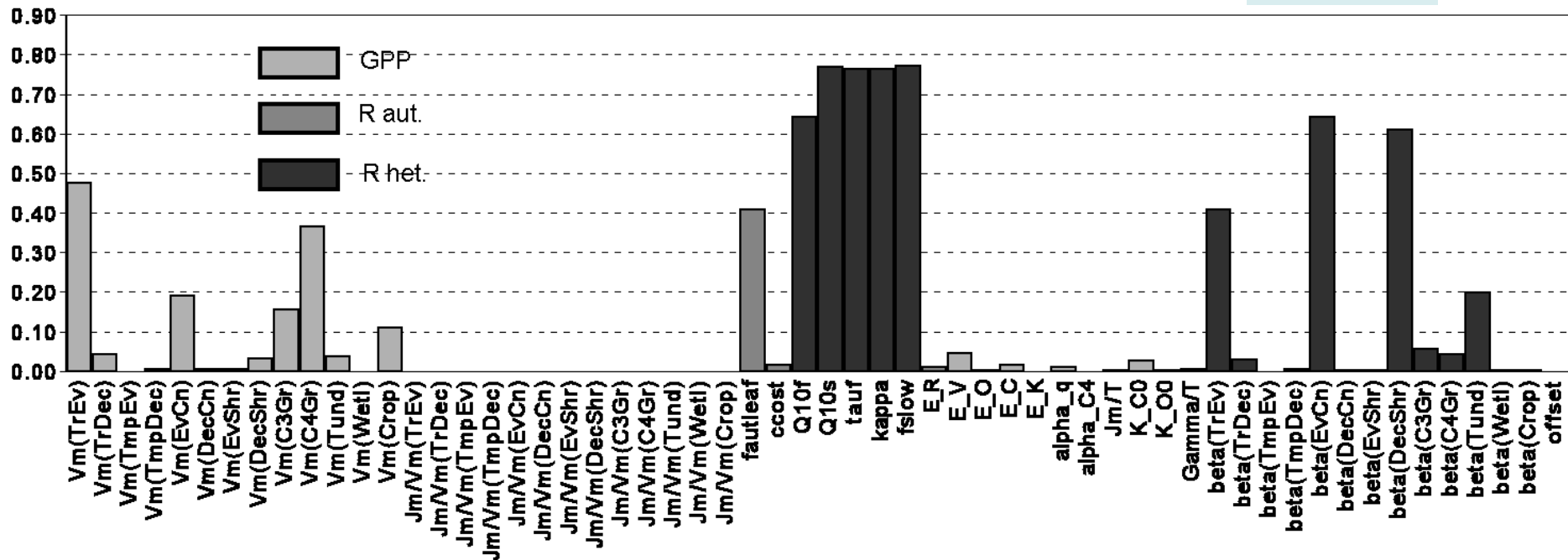
Figure taken from Tarantola '87

Space of  $m$  (model parameters)

# Parameter Uncertainties after optimisation

Relative reduction of parameter-uncertainty after optimisation:

$$1 - \frac{\sigma_{post}}{\sigma_{prior}}$$



# Propagating Uncertainties

The model (with optimised parameters) is used to compute diagnostics,  $y$ , (e.g. CO<sub>2</sub> fluxes):

$$y = y(m_{opt})$$

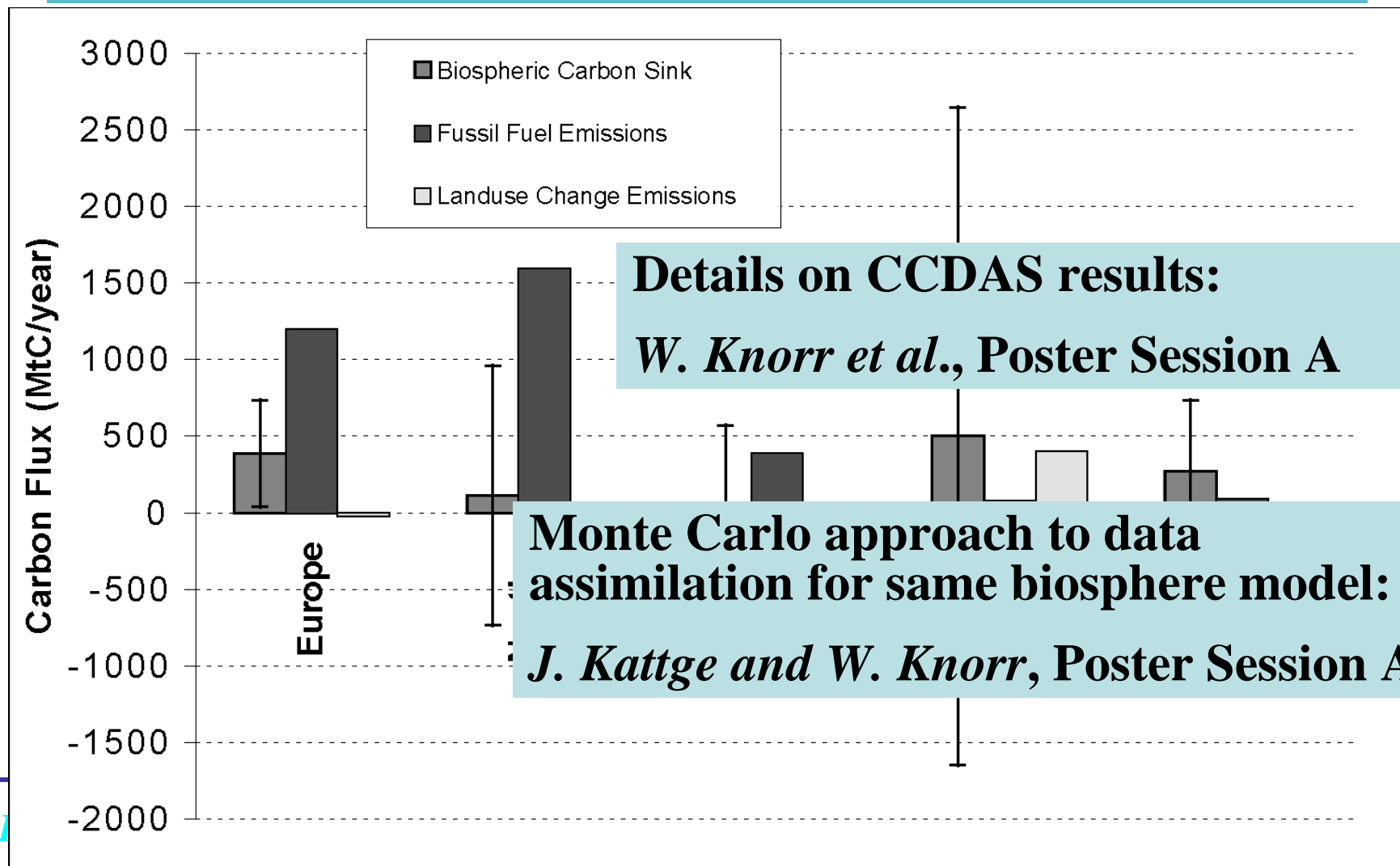
Covariance of uncertainty in  $y$ :

$$\mathbf{C}_y \approx \left( \frac{\partial y_i(m_{opt})}{\partial m_j} \right) \mathbf{C}_m \left( \frac{\partial y_i(m_{opt})}{\partial m_j} \right)^T$$

adjoint or  
tangent linear  
model

covariance  
of parameter uncertainties

# Regional Net Carbon Balance and Uncertainties



# Summary of CCDAS-Approach

- **CCDAS tests given combination of model formulation + set of observations + prior assumptions**
- **CCDAS delivers optimal parameters + uncertainties, and diagnostics/prognostics + uncertainties**
- **CCDAS allows fair comparison of different model formulations because it chooses optimal parameter set**
- **Sensitivities and comparison with observations are tough tests for the model and are providing many hints for model development**
- **Model is permanently improved within CCDAS, quick updates of derivative code via automatic differentiation (TAF)**
- **Numbers are based on version we were not yet happy with**

# Adjoint Applications Summary

## Shown

- Sensitivity Analysis
- Data Assimilation/Optimisation
- Uncertainty Propagation

## Not shown:

- Singular Vectors/Stability Analysis
- Targeted Observations

# Some automatically generated Adjointns of ESM-components

AGCMs:	PUMA, MM5, <b>fvGCM</b>
OGCMs:	HOPE, <b>MITgcm</b> , <b>MOM3</b>
Biosphere:	<b>BETHY</b>
CTMs:	<b>TM2</b> , <b>TM3</b>
Seaice:	<b>MITgcm</b>
Ocean waves:	<b>WAM</b>
Socioeconomics:	<b>SIAM</b>



# Summary

- **Adjoint support model development**
- **Automatic generation of adjoints reduces delays from model development to adjoint applications**
- **There are adjoints for many ESM components**

# Posters/Links

## Poster Session A (until 1pm tomorrow)

- *P. Heimbach + ECCO Consortium:*  
Maintaining Adjoint of MITgcm
- *W. Knorr et al.:* CCDAS results
- *J. Kattge and W. Knorr,* BETHY+Monte Carlo

- [www.CCDAS.org](http://www.CCDAS.org)
- [www.ecco-group.org](http://www.ecco-group.org)
- [www.FastOpt.com](http://www.FastOpt.com), at Hamburg, Schanzenstr. 36

# BETHY

## (Biosphere Energy-Transfer-Hydrology Scheme)

$\Delta\text{lat}, \Delta\text{lon} = 2 \text{ deg}$

- **GPP:**

C3 photosynthesis – *Farquhar et al. (1980)*

C4 photosynthesis – *Collatz et al. (1992)*

stomata – *Knorr (1997)*

$\Delta t = 1 \text{ h}$

- **R<sub>aut</sub>:**

maintenance respiration =  $f(N_{\text{leaf}}, T)$  – *Farquhar, Ryan (1991)*

growth respiration  $\sim$  NPP – *Ryan (1991)*

$\Delta t = 1 \text{ h}$

- **R<sub>het</sub>:**

fast/slow pool resp. =  $w^k Q_{10}^{T/10} C_{\text{fast/slow}} / \tau_{\text{fast/slow}}$

$\tau_{\text{slow}} \rightarrow \text{infin.}$

average NPP =  $\beta$  average R<sub>het</sub> (at each grid point)

$\Delta t = 1 \text{ day}$

$\beta < 1$ : source

$\beta > 1$ : sink

# Some larger TAF derivatives

Model (Who)	Lines	Lang	TLM	ADM	Ckp	HES
NASA/NCAR (w. Todling & Lin)	87'000	F90	2.7	6.8	2 lev	-
MOM3 (Galanti & Tziperman)	50'000	F77	Yes	4.6	2 lev	-
MITGCM (ECCO Consortium)	100'000	F77	1.8	5.5	3 lev	11.0/ 1
BETHY (w. Knorr, Rayner, Scholze)	5'500	F90	1.5	3.4	2 lev	50/ 12
Nav.-Stokes-Solver (Hinze, Slawig)	450	F77	-	2.0	steady	-
NSC2KE	2'500	F77	2.4	3.4	steady	9.8/ 1
HB_AIRFOIL (Thomas & Hall)	8'000	F90	-	3.0		-

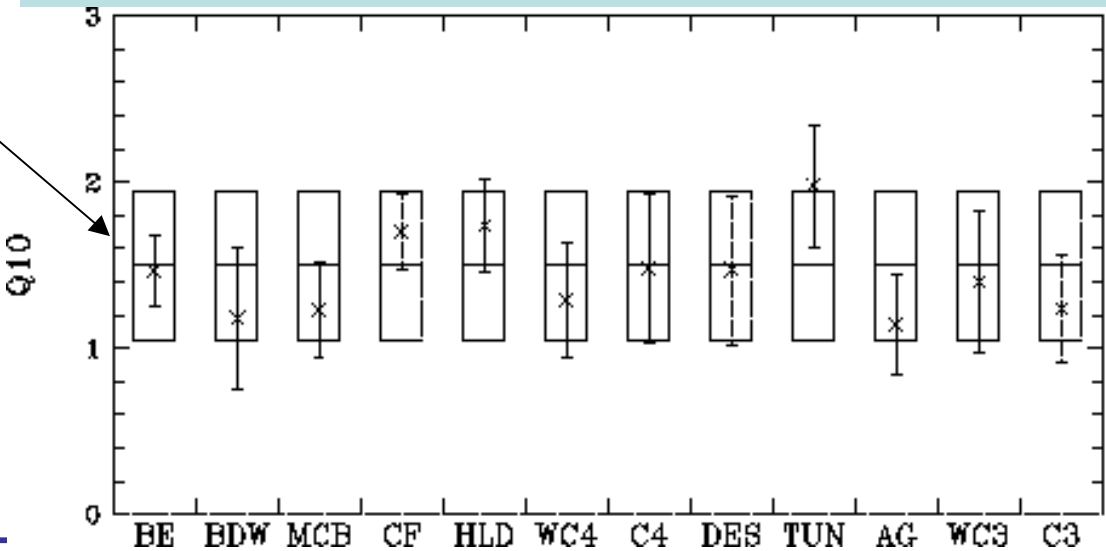
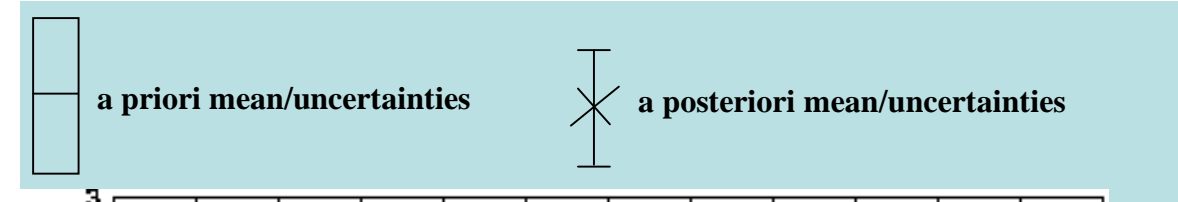
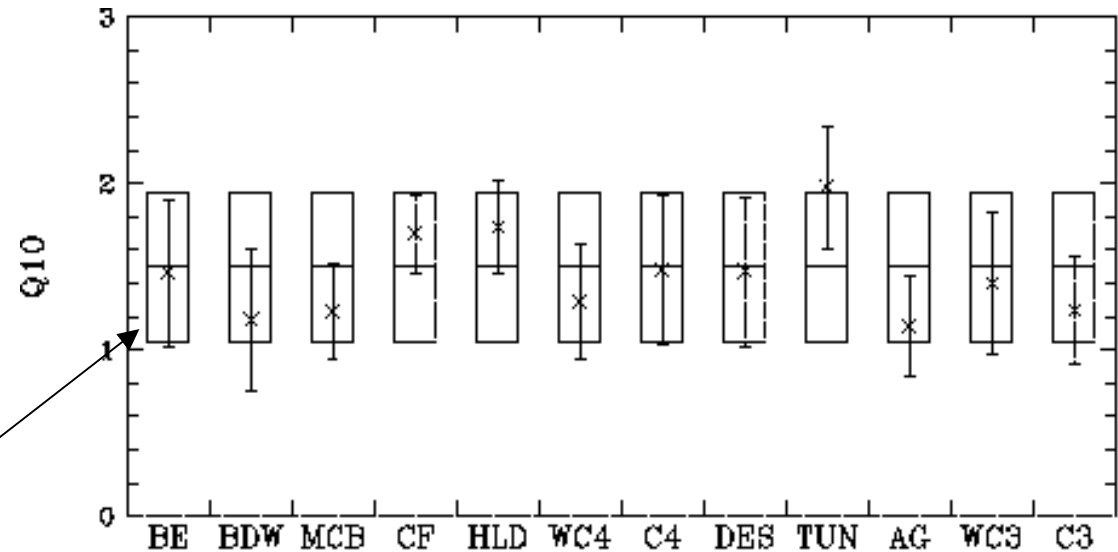
- **Lines:** total number of Fortran lines without comments
- **Numbers for TLM and ADM** give CPU time for (function + gradient) relative to forward model
- **HES format:** CPU time for Hessian \* n vectors rel. t. forw. model/ n
- **2 (3) level checkpointing** costs 1 (2) additional model run(s)

Comparison shows **impact of a (pseudo) flux measurement** in the **broadleaf evergreen biome** on Q10 estimated by an inversion of SDBM:

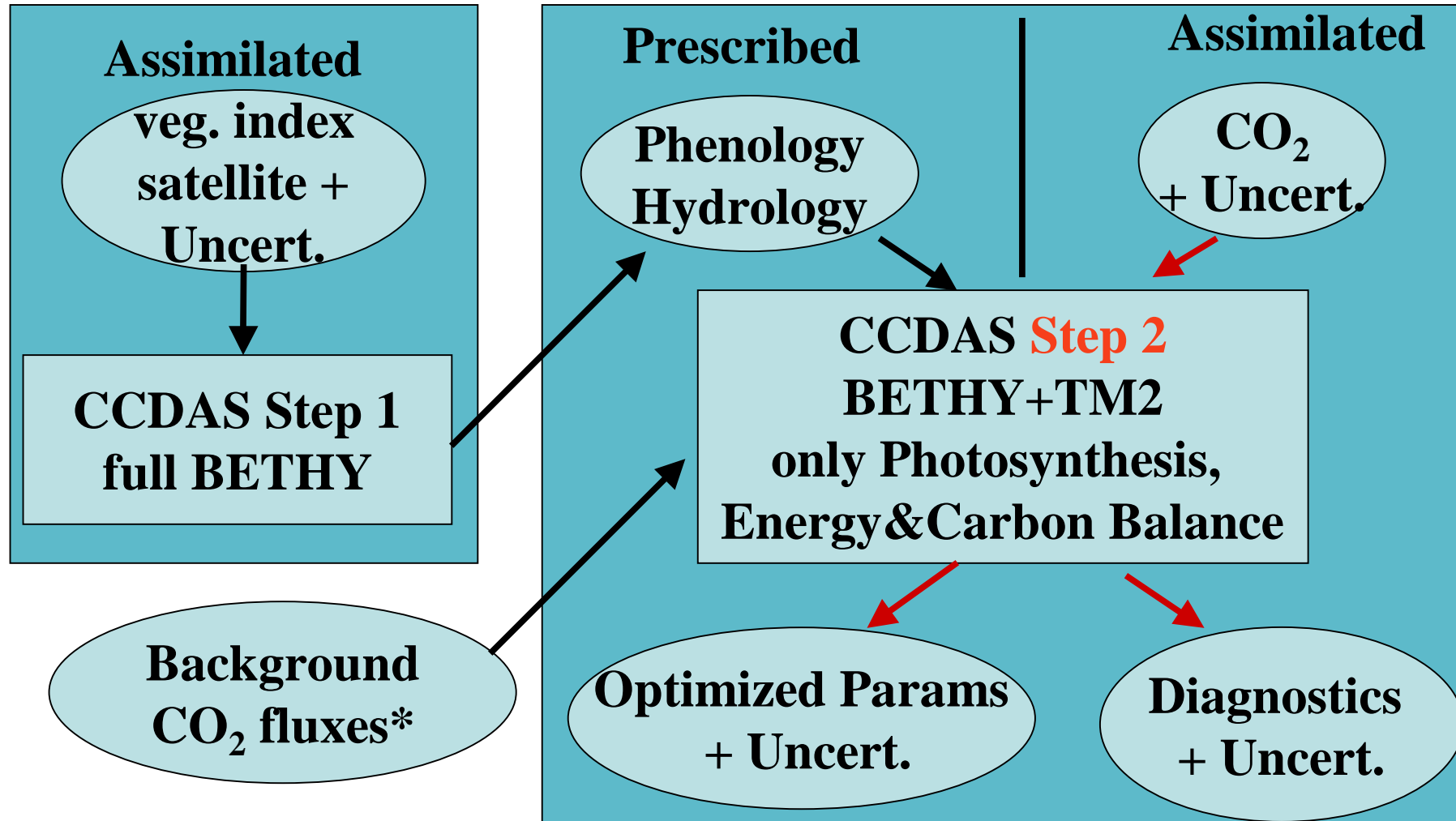
**Upper panel:**  
only concentration data

**Lower panel:**  
concentration data +  
pseudo flux measurement  
(mean: as predicted  
sigma:  $10\text{gC}/\text{m}^2/\text{year}$ )

Details:  
Kaminski et al., GBC, 2001



# Carbon Cycle Data Assimilation System (CCDAS)



\* ocean: Takahashi et al. (1999), LeQuere et al. (2000); emissions: Marland et al. (2001), Andres et al. (1996); land use: Houghton et al. (1990)